**PROCEEDINGS OF THE INSTITUTE OF VEHICLES 3(112)/2017**

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# SIMULATION MODEL OF LINEAR GENERATOR TEST BENCH

## 1. Introduction

Vehicle emissions and limited resources of fossil fuels make space for the development of ecological branches of the economy, especially in transportation. To reduce carbon footprint efficient stationary combustion processes or alternative energy sources can be used to generate electricity. Having in mind road vehicles both options are viable. Stationary combustion can be used as a power source in power plants or in combustion engines for vehicle on-board generators. At the other hand alternative energy sources like wind, water or solar can deliver energy to the grid and supply battery charging.

Because of the limited energy density of currently used electrochemical batteries and relatively long charging times, electric vehicle range is still a concern that negatively impacts the growth of the electric vehicle market. The solution to this problem is hybrid power train. By limiting the size of the electrochemical battery and applying novel types of internal combustion engines and electric generators a significant range extension can be obtained (compared to battery electric vehicles) and at other token fuel consumption and emission can be reduced (compared to internal combustion engine only drives). But hybrid electric vehicles cannot significantly reduce its emissions without efficient means of on-board power generation. Free piston engine combined with a linear generator has a number of positive characteristics that can justify its application in a new generation of extended-range electric vehicles, which can provide better efficiency than existing solutions [1, 2].

In this article, authors described the specific approach to modelling of the existing linear generator laboratory test bench, with detailed models of its components. It includes simulation models of mechanical and electrical components solved in the time domain. The goal is to obtain the possibility of estimation working parameters of the linear generator system, with use of the computer environment. The resultant virtual test bench gives flexibility for further investigation of linear generator parameters of operation.

## 2. Simulation model

### 2.1. Test bench structure

A range extending device based on the free piston engine converts chemical energy of fuel bonds to mechanical energy of a piston motion and then finally to electrical energy at generator output. The piston makes a reciprocating motion and so the mover of the generator connected directly to it. For providing this type of motion for the test bench generator mover, without using engine itself, a slider-crank mechanism propelled from the electric machine was applied.

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Figure 1 shows the test bench schematics. The main test subject is a linear generator (l. gen.) that charges the battery (batt.). Electric motor (~M) controller (inv.) is powered from the grid. Motor speed can be adjusted via control panel or by the personal computer (PC) and data acquisition/signal generator analog-digital converter card (I/O). The motor is connected to the linear generator mover by slider-crank mechanism (s. -c. m.). The mover has mounted NdFeB type permanent magnets. Permanent magnets magnetic field moving relative to the generator stator winding induces in the winding electromotive force. The flow of charge to the lithium-ion battery (batt.) is possible through application of rectifier (rectif.) and direct current converter (DC/-DC) that adjust required voltage and charging current. The test bench has a number of sensors, including rotary position sensor (p. s.), torque sensor (t. s.), voltmeters (V) and ammeters (A) which measurement data can be recorded online via data acquisition card (I/O) and the computer (PC).

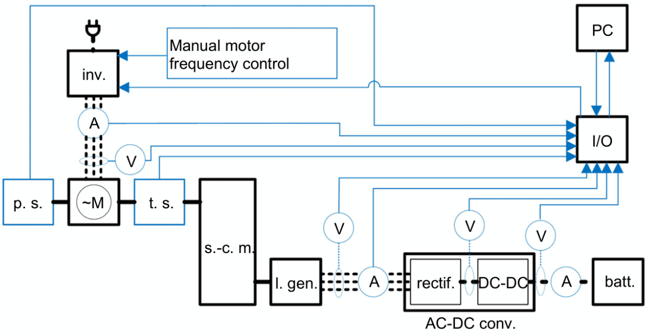

Fig. 1. Test bench structure schematics

## 2.2. Asynchronous motor and controller model

Three phase asynchronous motor (AC motor) model is made in d-q coordinates. D-q axes are revolving with rotor speed $\omega_s$ [3]. Equation 1-9 describe relations between voltages *u* and currents *i*, where $\phi$, *R*, *L* are magnetic fluxes, resistances, and inductances respectively. Subscripts *s*, *r*, ‘, are referring to stator parameters, rotor parameters and parameters transformed to the stator side respectively. Subscripts *l* and *m* are indicating leakage inductance and magnetizing inductance respectively. Voltages applied to the motor AC motor in *d* and *q* axis (eq. 10-11) result in produced value of electromagnetic torque $M_e$ and subsequently to motor speed $\omega$, which also depends on the motor inertia *J* and external torque load $M_{ext}$ (eq. 12).

$$u_{qs} = R_s i_{qs} + \frac{d\phi_{qs}}{dt} + \omega_s \phi_{ds} \tag{1}$$

$$u_{ds} = R_s i_{ds} + \frac{d\phi_{ds}}{dt} + \omega_s \phi_{qs} \tag{2}$$

$$u'_{qr} = R'_r i'_{qr} + \frac{d\phi'_{qr}}{dt} + (\omega_s - \omega_e)\phi'_{dr} = R'_r i'_{qr} + \frac{d}{dt}\left(L'_r i'_{qr} + L_m i_{qs}\right) + (\omega_s - \omega_e)\left(L'_r i'_{dr} + L_m i_{ds}\right) \tag{3}$$

$$u'_{dr} = R'_r i'_{dr} + \frac{d\phi'_{dr}}{dt} + (\omega_s - \omega_e)\phi'_{qr} = R'_r i'_{dr} + \frac{d}{dt}\left(L'_r i'_{dr} + L_m i_{ds}\right) + (\omega_s - \omega_e)\left(L'_r i'_{qr} + L_m i_{qs}\right) \tag{4}$$

$$M_e = \frac{3}{2}\left(\phi_{ds} i_{qs} - \phi_{qs} i_{ds}\right) = \frac{3}{2}\left[\left(L_s i_{ds} + L_m i'_{dr}\right) i_{qs} - \left(L_s i_{qs} + L_m i'_{qr}\right) i_{ds}\right] \tag{5}$$

$$L_s = L_{ls} + L_m \tag{6}$$

$$L'_r = L'_{lr} + L_m \tag{7}$$

$$\omega_s = 2\pi\, f \tag{8}$$

$$\omega_e = \omega p \tag{9}$$

$$u_{qs} = U_m \cos(2\pi\, ft) \tag{10}$$

$$u_{ds} = U_m \sin(2\pi\, ft) \tag{11}$$

$$\omega = \int \frac{M_e - M_{ext}}{J} dt + \omega_0 \tag{12}$$

Motor stator winding has *p* magnetic pole pairs. Supplied voltage of frequency *f* (eq. 10-11) is controlled via the motor controller. The controller delivers to the motor three phase sinusoidal voltages shifted by an angle of π/3 rad. This voltages via pulse-width modulation. Comparing three sine waves of desired frequency and amplitude from the range of -1 and 1, with a sawtooth wave of the same range but much higher frequency (switching frequency) generate signals for enabling power transistors. Transistors output DC link voltage of value $U_m$ at specific moments and with desired polarisation. The controller operates with respect to condition *U/f=const.*, therefore maximal torque can be constant in the whole range of motor speeds.

### 2.3. Mechanical model with slider-crank mechanism

Mechanical setup is made of the motor rotor connected with slider-crank mechanism and mover, as in Figure 2.

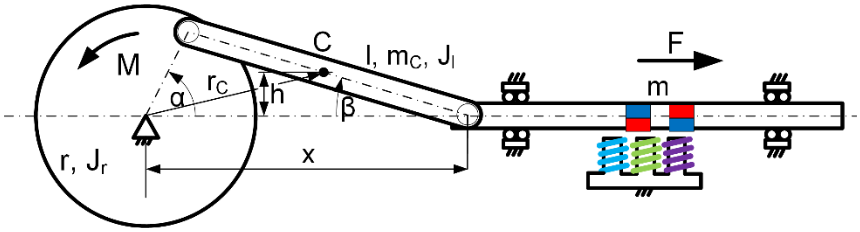

Fig. 2. Mechanical setup of laboratory test bench

For obtaining waveforms of forces, position, speed, and acceleration as a time functions, the motion of presented setup was described in a function of coordinate α, with Lagrange’s equations of motion. The kinetic energy of this system is given by equation 13, with equations 14-18 and 19-23 for specific variables of position and speed.

$$E_k = \frac{1}{2} J_r \dot{\alpha}^2 + \frac{1}{2} m_C \dot{r}_C^{\,2} + \frac{1}{2} J_l \dot{\beta}^2 + \frac{1}{2} m \dot{x}^2 \tag{13}$$

$$\beta = \arcsin\left( \frac{r}{l} \sin\alpha \right) \tag{14}$$

$$x = r\cos\alpha + l\cos\beta \tag{15}$$

$$r_{Cx} = r\cos\alpha + \frac{l}{2}\cos\beta \tag{16}$$

$$r_{Cy} = \frac{r}{2}\sin\alpha \tag{17}$$

$$r_C = \sqrt{r_{Cx}^2 + r_{CY}^2} \tag{18}$$

$$\dot{\beta} = \frac{1}{\sqrt{1 - \frac{r^2}{l^2}(\sin\alpha)^2}} \frac{r}{l} \cos\alpha\, \dot{\alpha} \tag{19}$$

$$\dot{x} = \sqrt{r^2 \dot{\alpha}^2 (\sin\alpha)^2 \left(1 + 2b + b^2\right)} \tag{20}$$

$$\dot{r}_C = \sqrt{r^2 \dot{\alpha}^2 \left[ (\sin\alpha)^2 \left( \frac{3}{4} + b + \frac{1}{4} b^2 \right) + \frac{1}{4} \right]} \tag{21}$$

$$b = c^{-\frac{1}{2}} \frac{r}{l} \cos\alpha \tag{22}$$

$$c = 1 = \frac{r^2}{l^2} (\sin\alpha)^2 \tag{23}$$

Potential energy varies only for connecting rod (eq. 24).

$$E_p = m_C g \frac{r}{2} \sin\alpha \tag{24}$$

The system has one degree of freedom, with generalized force $Q_\alpha$ acting on it (eq. 25).

$$\frac{d}{dt}\left( \frac{dE_k}{d\dot{\alpha}} \right) - \frac{dE_k}{d\alpha} + \frac{dE_p}{d\alpha} = Q_\alpha \tag{25}$$

Derivatives of kinetic energy over generalized speed and time are calculated (eq. 26-29).

$$\frac{d}{dt}\left[ \frac{d}{d\dot{\alpha}} \left( \frac{1}{2} J_r \dot{\alpha}^2 \right) \right] = J_r \ddot{\alpha} \tag{26}$$

$$\frac{d}{dt}\left[\frac{d}{d\dot{\alpha}}\left(\frac{1}{2}m_C\,\dot{r}_C^{\,2}\right)\right]=m_C r^2\left\{\begin{array}{l}\ddot{\alpha}\left[(\sin\alpha)^2\left(\frac{3}{4}+b+\frac{1}{4}b^2\right)+\frac{1}{4}\right]+\\+\dot{\alpha}^{2}(\sin 2\alpha)\left(\frac{3}{4}+b+\frac{1}{4}b^2\right)+\\+\dot{\alpha}^{2}(\sin\alpha)^2\frac{r}{l}\sin\alpha\left[\left(\frac{r}{l}\cos\alpha\right)^2 c^{-\frac{3}{2}}-c^{-\frac{1}{2}}\right]\end{array}\right\} \tag{27}$$

$$\frac{d}{dt}\left[\frac{d}{d\dot{\alpha}}\left(\frac{1}{2}J_l\,\dot{\beta}^{2}\right)\right]=J_l\left[\ddot{\alpha}^{2}\,b^2+2\dot{\alpha}^{2}\,b\frac{r}{l}\sin\alpha\left(c^{-\frac{3}{2}}\frac{r^2}{l^2}\cos^2\alpha-c^{-\frac{1}{2}}\right)\right] \tag{28}$$

$$\frac{d}{dt}\left[\frac{d}{d\dot{\alpha}}\left(\frac{1}{2}m\,\dot{x}^{2}\right)\right]=mr^2\left\{\begin{array}{l}\ddot{\alpha}\left[(\sin\alpha)^2\left(1+2b+b^2\right)\right]+\dot{\alpha}^{2}\sin 2\alpha\left(1+2b+b^2\right)+\\+\dot{\alpha}^{2}\left[(\sin\alpha)^2 2(1+b)\frac{r}{l}\sin\alpha\left(c^{-\frac{3}{2}}\frac{r^2}{l^2}\cos^2\alpha-c^{-\frac{1}{2}}\right)\right]\end{array}\right\} \tag{29}$$

Derivatives of kinetic energy over generalized coordinate α are calculated (eq. 30-34).

$$\frac{d}{d\alpha}\left(\frac{1}{2}J_r\,\dot{\alpha}^{2}\right)=0 \tag{30}$$

$$\frac{d}{d\alpha}\left(\frac{1}{2}m_C\,\dot{r}_C^{\,2}\right)=\frac{1}{2}m_C r^2\,\dot{\alpha}^{2}\left\{\sin 2\alpha\left(\frac{3}{4}+b+\frac{1}{4}b^2\right)+(\sin\alpha)^2\left[\left(1+\frac{1}{2}b\right)b'^{\alpha}\right]\right\} \tag{31}$$

where:

$$b'^{\alpha}=\frac{r}{l}\left(\frac{1}{2}c^{-\frac{3}{2}}\frac{r^2}{l^2}\sin 2\alpha-c^{-\frac{1}{2}}\sin\alpha\right) \tag{32}$$

$$\frac{d}{d\alpha}\left(\frac{1}{2}J_l\,\dot{\beta}^{2}\right)=J_l\,\dot{\alpha}^{2}\,bb'^{\alpha} \tag{33}$$

$$\frac{d}{d\alpha}\left(\frac{1}{2}m\,\dot{x}^{2}\right)=\frac{1}{2}mr^2\,\dot{\alpha}^{2}\left\{\sin 2\alpha\left(1+2b+b^2\right)+(\sin\alpha)^2\left[2b'^{\alpha}(1+b)\right]\right\} \tag{34}$$

A derivative of potential energy over generalized coordinate α is calculated (eq. 35).

$$\frac{dE_p}{d\alpha} = m_c g \frac{r}{2} \cos\alpha \tag{35}$$

Generalized force, when the force of friction is omitted, consist of motor torque *M* and generator force *F*. Generator force is braking force created by the combined effect of interacting magnetic fields of stator and mover (eq. 36).

$$Q_\alpha = M - Fr\sin\alpha\left(1 + c^{-\frac{1}{2}}\frac{r}{l}\cos\alpha\right) \tag{36}$$

## 2.4. Linear generator model

Mover position over time is an input variable in the generator model. Generator setup here has one magnetic pole pair (two magnets) of mover and three phase ($n \in \{1,2,3\}$) stator winding wound over three stator teeth. This setup can be further expanded by adding multiple magnet pairs and/or stator modules. Electromotive force value $e_n$ induced in the *n*-th coil is calculated as derivative of magnetic flux linkage of this coil over time (flux distribution is further discussed in [4]). Its voltage $u_n$ depends on voltage drop caused by current *i* flowing through the coil. Created force *F* is a result of coil current flow in moving magnetic field. Generator model is described by the equations 37-39.

$$e_n = -\frac{d\psi_n(t)}{dt} \tag{37}$$

$$u_n(t) = e_n(t) - Ri_n(t) - L\frac{di_n(t)}{dt} \tag{38}$$

$$F_n(x,t) = i_n(t)\left(-\frac{d\psi_n(x)}{dx}\right) \tag{39}$$

where: $R$, $L$, $\Psi_n$ – *n*-th coil resistance, inductance, and flux linkage respectively.

## 2.5. Power converter model

Electric schematic of the test bench is shown in Figure 3.

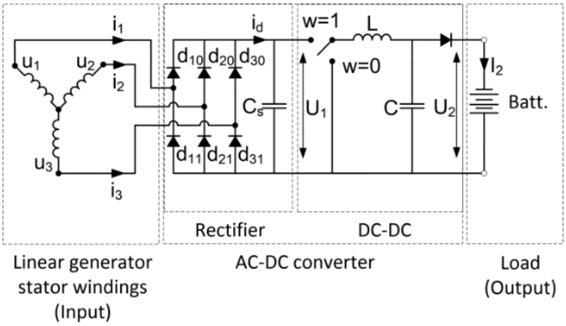

Fig. 3. Schematic of test bench electrical connections

Stator three-phase winding is star connected and feed electricity to the three phase rectifier. DC-DC converter lowers the voltage to the appropriate level for battery charging and can control charging current value as well. Diode H-bridge and capacitor $C_s$ provide smooth DC voltage $U_1$ to the DC-DC converter block. Switch $w$ represents power transistor. When the switch is on ($w$=1) capacitor $C$ is charged. When the switch is off ($w$=0) capacitor $C$ charges the battery. Inductance $L$ smooth out current waveform. By changing on and off switching frequency, output voltage $U_1$ is adjusted. Because of the complexity of mathematical description of this electric circuit and inclusion of discontinuous states provided by electronic components, model was made in Simulink Simscape environment. This specialised piece of software exist as Matlab toolbox and is dedicated to solving electrical circuits by including premade component blocks, such as resistors, capacitors, inductors, diodes and transistors.

## 2.6. Lithium-ion battery model

Lithium-ion battery model was made based on equivalent circuit diagram consisting of nonlinear electromotive force source $e$ and nonlinear internal resistance $R_w$ [5, 6] expressed as polynomials with coefficients $A$ and $B$ respectively. Parameters of these elements are functions of state of charge $k$ (eq. 40-43). Voltage of the battery is function of $k$ and current $I_2$. The battery is made from $n_c$ cells connected in series.

$$e(k) = \sum_{jw=0}^{nw} A_{jw} k^{jw} \tag{40}$$

$$R_w(k) = \sum_{jw=0}^{nw} B_{jw} k^{jw} \tag{41}$$

$$U_2(k,i) = n_c \left[ e(k) - R_w(k) I_2 \right] \tag{42}$$

$$k = k_0 + \frac{1}{Q_n} \int I_2(t)dt \tag{43}$$

State of charge $k$, internal resistance $R_w$ and battery current $I_2$ are time functions and describe momentary battery state. Nominal battery capacity $Q_n$ is assumed to be constant.

# 3. Simulation test

## 3.1. Test bench parameters

Computer simulation of virtual test bench was made in Matlab Simulink, with usage of Simscape toolbox for electric circuit model. Values of parameters of the simulation model are presented in the tables 1 and 2, and other linear generator parameters are the same as presented in [4].

Table 1. Parameters of the simulation model

| Parameters | Symbol | Value |
|---|---|---|
| AC motor nominal power | $P_m$ | $4\cdot 10^3$ W |
| Motor rated speed | $n_m$ | 1430 rpm |
| Stator resistance | $R_s$ | 1.4 Ω |
| Rotor resistance | $R'_r$ | 1.4 Ω |
| Stator inductance | $L_s$ | $6\cdot 10^{-3}$ H |
| Rotor inductance | $L'_r$ | $6\cdot 10^{-3}$ H |
| Magnetizing inductance | $L_m$ | 0.17 H |
| Motor rotor inertia | $J$ | 0.013 $\text{kg}\cdot\text{m}^2$ |
| Generator mover mass | $m$ | 0.5 kg |
| Crank throw | $r$ | 0.03 m |
| Connecting rod length | $l$ | 0.3 m |
| Crank inertia | $J_r$ | 0.005 $\text{kg}\cdot\text{m}^2$ |
| Connecting rod mass | $m_C$ | 0.15 kg |
| Connecting rod inertia (over C) | $J_l$ | 0.001 $\text{kg}\cdot\text{m}^2$ |
| Converter capacitors capacitance | $C$ | $1.5\cdot 10^{-3}$ F |
| Converters inductance | $L$ | $L=1\cdot 10^{-4}$ H |
| Battery initial state of charge | $k_0$ | 0.5 |

Table 2. *A* and *B* coefficients for the MGL 90 Ah battery model

| *jw* | $A_{jw}$ | $B_{jw}$ |
|---|---|---|
| 0 | 3.51 V | 0.00112 Ω |
| 1 | 1.62 V | -0.00117 Ω |
| 2 | -2.23 V | 0.00213 Ω |
| 3 | 1.26 V | -0.00128 Ω |

### 3.2. Test bench parameters simulation

After setting desired motor speed to 600 rpm and battery current $I_2$ to 20 A in simulation, we simulated motor speed $\omega$ and generator force $F$ (Fig. 4) along with battery voltage $U_2$ and battery state of charge $k$ (Fig. 5) at the time span of 1 s.

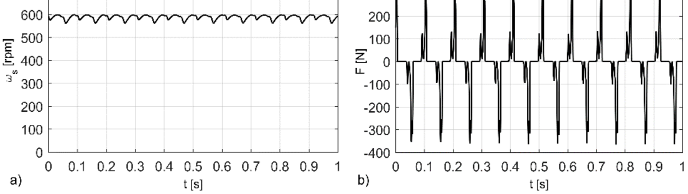

Fig. 4. Results of the linear generator simulation (part 1):
a) momentary motor speed; b) momentary generator force

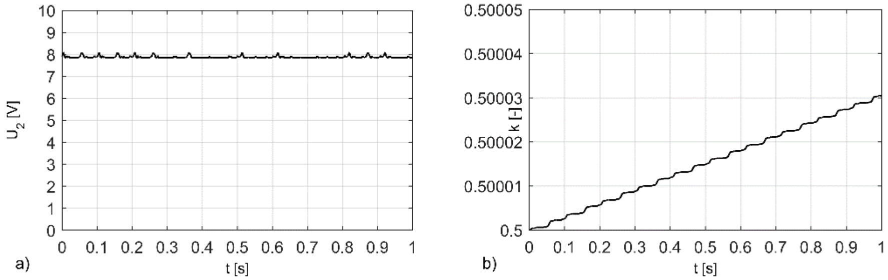

Fig. 5. Results of the linear generator simulation (part 2):
a) momentary battery voltage; b) momentary battery state of charge

Results show that due to combined effects of the generator force changes during oscillatory motion (Fig. 4b) and effects of inertia forces, the resultant AC motor speed will show some deviation from set value (Fig. 4a). Battery charging process will proceed smoothly due to the steady output voltage (Fig. 5a) and battery state of charge will proceed to raise at a steady pace (Fig. 5b).

## 4. Conclusions

By application of properly formulated simulation models of linear generator test bench components, resultant complex simulation model was designed. Mathematical models of the components were created with a usage of differential equations and then integrated into computer software to create combined model. Solution to this model provides values of time depending test bench parameters and some of them were presented in the paper as simulation results. Solved model can also provide any other time function value that was implemented as a variable. This is an advantage of virtual test bench over real test bench since some of the parameters are difficult to measure in real-world conditions. But there is a trade-off. Modelling process implements some level of simplification, both in terms of equations complexity and equations coefficients values accuracy. Therefore further simulation result verification and model correction is needed. A such model is a useful tool for a fast testing of a behaviour of the linear generator operating in the conditions similar for that expected in the range extended electric vehicle.

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## Abstract

Electric vehicles require usage of economical and ecological solutions of range extenders with internal combustion engines. One of the proposed solutions can be an free piston combustion engine paired with a linear generator. Authors proposed simulation model for the laboratory test bench, which purpose is to be used for testing of permanent magnet linear generator. Mathematical models of three phase asynchronous motor with controller, slider-crank mechanism, permanent magnet linear generator, DC-DC converter and lithium-ion battery were used. The complex simulation model gives an opportunity for assessing dynamic time-based characteristics of test stand before manufacturing or to use for batch testing, giving large time savings on the test stand implementation and tests execution. Exemplary results of simulation solved in the Matlab Simulink software are presented, for chosen input parameters values.

**Keywords:** linear generator, simulation model, electric drive

## MODEL SYMULACYJNY STANOWISKA Z GENERATOREM LINIOWYM

## Streszczenie

Samochody elektryczne, w celu zwiększenia ich autonomiczności jazdy, wymagają zastosowania ekonomicznych i ekologicznych rozwiązań generatorów elektrycznych z silnikami spalinowymi. Jednym z takich rozwiązań może być silnik spalinowy typu „free piston” współpracujący z generatorem liniowym. Zaproponowano model symulacyjny projektu stanowiska badawczego do testowania generatora liniowego z magnesami trwałymi. Skorzystano przy tym z modeli matematycznych głównych komponentów: trójfazowego silnika asynchronicznego wraz ze sterownikiem, układu korbowo-wodzikowego, elektrycznego generatora liniowego z magnesami trwałymi, przekształtnika energii elektrycznej oraz baterii litowo-jonowej. Kompleksowy model może być wykorzystany do oceny parametrów pracy stanowiska oceny przed jego wykonaniem lub użyty do przeprowadzania wielokrotnych testów badawczych, co znacznie skraca czas wdrożenia i czas badań. Przedstawiono przykładowe wyniki symulacji zaimplementowanej do środowiska komputerowego Matlab Simulink, dla zadanych wartości parametrów zadanych.

**Słowa kluczowe:** generator liniowy, model symulacyjny, napęd elektryczny